

SHEAR FLOW OF ELASTOVISCOUS MEDIUM ON SELF-HEATING  
BETWEEN TWO DISKS

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With an accuracy sufficient for engineering applications, the problem of motion of an elastoviscous liquid between disks with self-heating is solved. The theoretical dependences are compared with experimental data for the normal stress.

1. The problem is considered in connection with the need to calculate the motion of polymer liquid in the seal of a rotating axle of new type [1], the action of which is based on normal stress. In addition, it may be used to investigate disk extrusion [2].

As a preliminary, the experimental apparatus for studying the motion of a polymer medium between disks with simultaneous measurement of the radial normal stress is discussed (Fig. 1). The polymer medium 1 is in a gap of thickness  $H = 2h$  ( $H = 1.7 \cdot 10^{-3}$  m) between disks 2 and 3 of radius  $R_1$ . The lower disk rotates at constant angular velocity  $\Omega$ ; the upper disk is motionless and rigidly connected to the cylindrical frame 4. In disk 3, there is an aperture of radius  $R_2$  ( $R_2 = 10^{-3}, 7 \cdot 10^{-3}, 1.5 \cdot 10^{-2}$  m;  $R_1 = 4 \cdot 10^{-2}$  m). Part of the frame volume is also filled with polymer medium 5, which is practically undeformed. This serves to isolate the polymer deformed in the gap 1 from the water 6 which fills the remainder of the cylindrical frame (in the presence of water, breakaway of the deformed polymer medium from the wall is possible [1]). In disk rotation in the polymer medium, as well as tangential stress, radial stress develops, in particular; its action is transmitted through the aperture to the motionless water, the pressure  $P_1$  in which is measured by manometer 7.

The influence of the velocity of disk rotation  $\Omega$  and the geometric dimensions  $R_2, h$  on  $P_1$  is experimentally investigated in the present work.

If  $R_2 \sim h \ll R_1$ , the influence of the aperture on the motion between the disks may be disregarded. When  $R_2 > h$  ( $R_2 = 7 \cdot 10^{-3}$  m and  $1.5 \cdot 10^{-2}$  m,  $H = 1.7 \cdot 10^{-3}$ ), the contribution of the neutral region to the experimentally measurable pressure  $P_1$  is eliminated. To this end, the (exploratory) experiments are conducted both with complete filling of the gap between the disks with polymer and with filling only of the neutral zone.

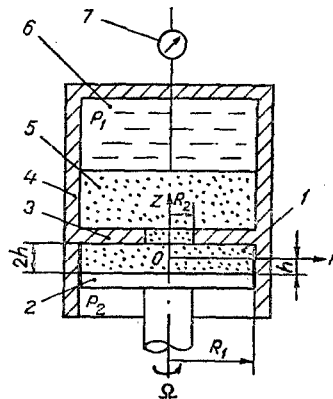


Fig. 1. Experimental apparatus.

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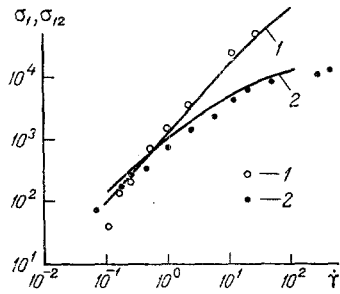


Fig. 2

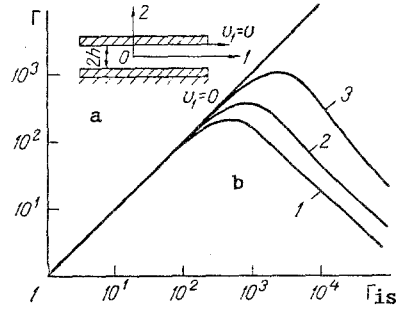


Fig. 3

Fig. 2. Flow curve and first difference of normal stress (1) for a melt of polydisperse polyisobutylene with  $MM \approx 5.7 \cdot 10^3$  in isothermal flow; the curves are theoretical dependences.  $\sigma_1, \sigma_{12}$ , N,  $m^2$ ;  $\dot{\gamma}$ ,  $sec^{-1}$ .

Fig. 3. Shear between two parallel plates: a) shear scheme; b) dependence of the true dimensionless shear velocity  $\Gamma$  on the shear velocity of isothermal motion  $\Gamma_{is}$  in the motion of a system with self-heating for different values of the parameter  $\Pi$ : 1)  $5.47 \cdot 10^{-5}$ ; 2)  $2.19 \cdot 10^{-5}$ ; 3)  $4.92 \cdot 10^{-5}$ .

The temperature of the disk 3 and the frame 4 are maintained constant. To this end, the disk 3 and frame 4 are hollow, to permit the circulation of cooling liquid; disk 2 is either cooled by free liquid jets (temperature of the disk as in disk 3) or is heat-insulated.

A polyisobutylene melt with molecular mass  $MM = 5.7 \cdot 10^3$  and the greatest newtonian viscosity  $\eta \sim 10^3$  Pa·sec (at a temperature  $T_0 = 20^\circ C$ ) is investigated. The flow curve  $\sigma_{12}(\dot{\gamma})$  and the first normal-stress difference  $\sigma_1(\dot{\gamma})$  obtained experimentally with a simple steady shift ( $20^\circ C$ ) for this melt are shown by points in Fig. 2.

2. First consider the problem of inertialess steady flow of an elastoviscous medium between parallel plates of infinite length in conditions of self-heating. This problem was solved for a power-law liquid in [3], and for a Maxwellian single-mode medium in [4].

Suppose that the velocity of the upper plate  $v_1 = v = \text{const}$ , and the lower plate is motionless. The temperature at the upper and lower plates  $T = T_0 = \text{const}$ . The motion is investigated in the rectangular coordinate system  $Ox_1x_2x_3$ . The coordinate origin (Fig. 3a) is placed in the center of the gap, the width of which is  $H = 2h$ . Axis 1 is directed along the motion and axis 2 perpendicular to the plate.

In flow with self-heating of the system, it is assumed that the shear flow at each point of the medium is conserved. In this case, the shear velocity  $\dot{\gamma}$  is not constant, as in the isothermal case, but depends on the coordinate  $x_2$ .

The stress components in shear are calculated on the basis of a version of the Maxwellian model [5]. Connecting  $N$  such models in parallel, it is found that

$$\begin{aligned} \sigma_1 &= \sigma_{11} - \sigma_{22} = 4 \sum_{h=1}^N (\mu_h/n) f(c_h) (c_h - c_h^{-1}), \\ \sigma_2 &= \sigma_{22} - \sigma_{33} = 2 \sum_{h=1}^N (\mu_h/n) [-2 + (c_h^{n/2} + c_h^{-n/2}) - f(c_h) (c_h - c_h^{-1})], \\ \sigma_{12} &= 4 \sum_{h=1}^N (\mu_h/n) f(c_h), \quad f(c_h) = (c_h^{n/2} - c_h^{-n/2}) / (c_h - c_h^{-1}), \\ c_h^2 - c_h^{-2} &= 4\Gamma_h, \quad \Gamma = \dot{\gamma}\theta_1, \quad \Gamma_h = \beta_h\Gamma, \quad \beta_h = \theta_h/\theta_1, \end{aligned} \tag{1}$$

where  $\beta_k$  does not depend on the temperature, according to the principle of temperature-time superposition [6].

The heat-conduction equation with steady shear

$$\tilde{\kappa} \frac{\partial^2 T}{\partial x_2^2} + \sigma_{12}(\dot{\gamma})\dot{\gamma} = 0 \quad (2)$$

relates the shear velocity  $\dot{\gamma}$  and the temperature  $T$ . Note that the heat source  $\sigma_{12}\dot{\gamma}$  (in the general case,  $\text{tr}\sigma \cdot \mathbf{e}$ ) remains the same in nonsteady flow [7]. This does not lead to instability of the steady Couette flow, in contrast to the case  $\text{tr}\sigma \cdot \mathbf{e}_p$  [8].

According to the definition of the shear velocity

$$\dot{\gamma} = \frac{dv_1}{dx_2} \quad (3)$$

Suppose that the relative temperature increment in self-heating of the system is  $\Delta T/T_0 \ll 1$ . This condition is usually significant because of the thermodestruction of polymers. Then the dependence of the constants of the medium on the temperature is as follows [6]

$$\theta_k = \theta_{(0)k} \exp[-m(T - T_0)], \quad \mu = \text{const}, \quad m = E/RT_0^2 \quad (4)$$

Equations (1)-(4), with the above boundary conditions, describe the isothermal motion of a medium with self-heating in steady shear.

The solution of the problem is sought under the assumption that the dimensionless deformation rate  $\Gamma$  is constant with fixed  $T_0$  and  $v$ . If the dimensionless quantities  $T^* = m(T - T_0)$ ,  $\xi = x_2/h$ ,  $u = v_1\theta_{(0)1}/\Gamma h$ ,  $\delta = \sigma_{12}(\Gamma_1\beta_k)\Gamma mh^2/(\tilde{\kappa}\theta_{(0)1})$ , are introduced, then from [3, 4, 9]

$$T^* = \ln \tilde{a} - \ln [\text{ch}^2(\xi \sqrt{\tilde{a}\delta/2})], \quad (5)$$

$$u = \sqrt{2\tilde{a}/\delta} [\text{th}(\xi \sqrt{\tilde{a}\delta/2}) + \text{th} \sqrt{\tilde{a}\delta/2}], \quad (6)$$

$$\tilde{a} = \text{ch}^2 \sqrt{\tilde{a}\delta/2}, \quad (7)$$

where  $\tilde{a}$  is the constant of integration.

Since when  $\xi = 1$  the dimensionless velocity  $u = 2\Gamma_{is}/\Gamma$  ( $\Gamma_{is} = v\theta_{(0)1}/2h$  is the dimensionless shear velocity in isothermal flow), it follows from Eq. (6) that

$$\Gamma_{is} = \Gamma \sqrt{2\tilde{a}/\delta} \text{th} \sqrt{\tilde{a}\delta/2} \quad (8)$$

It follows from Eq. (8) that  $\Gamma_{is}$  is a function of  $\Gamma$ , since  $\delta$  and  $\tilde{a}$  depend on  $\Gamma$ . Thus, from the specified  $\Gamma_{is}$ , Eq. (8) yields the unique value of the dimensionless deformation rate  $\Gamma$ , realized in flow with self-heating. Since  $\tilde{a}(\delta)$  is two-valued - see Eq. (7) - each value of  $\Gamma$  corresponds to two values of  $\Gamma_{is}$ . Thus, with the same stress state in Eq. (1), there are two different temperature - Eq. (5) - and velocity - Eq. (6) - profiles.

In Fig. 3b, the dependence of  $\Gamma$  on  $\Gamma_{is}$  is shown for different values of the dimensionless parameter  $\Pi = 2\mu_1 mh^2/\tilde{\kappa}\theta_{(0)1}$ . The relation between  $\delta$  and  $\Pi$  is as follows ( $\alpha_k = \mu_k/\mu_1$ ,  $\beta_k = \theta_k/\theta_1$ )

$$\delta = \frac{\sigma_{12}(\Gamma, \beta_k, \alpha_k, n)}{2\mu_1} \Pi \quad (9)$$

For each value of  $\Pi$ , there is a maximum possible value  $\Gamma = \Gamma_m$ , which decreases with increase in  $\Pi$ . The straight line in Fig. 3b corresponds to isothermal conditions of motion. It is evident from Fig. 3b that, with reduction in  $\Pi$  (reduction, for example, in the gap between the planes  $2h$ ), there is increase in the width of the range of  $\Gamma$ , where  $\Gamma \approx \Gamma_{is}$  and self-heating is not significant. In the region of significant self-heating with fixed  $\Pi$ , the dependence of  $\Gamma$  on  $\Gamma_{is}$  decreases. According to Eq. (1), all the stress components also decrease with increase in  $\Gamma_{is}$  in this region.

It follows from Eqs. (5) and (6) that

$$\xi = 0 \quad \frac{\partial T}{\partial \xi} = 0, \quad v_1(0) = \frac{v}{2} \left( u(0) = \frac{u}{2} \right) \quad (10)$$

The solution in Eqs. (5) and (6) with the condition in Eq. (10) corresponds to the following shear motion: the plate with the coordinate  $\xi = -1$  is immobile and thermostatted ( $T = T_0$ ); the plate with the coordinate  $\xi = 0$  is heat-insulated, and moves at a velocity  $v/2$ . The distance between the plates is  $h$ .

3. For theoretical consideration of the motion between two disks, the cylindrical coordinate system  $r, \varphi, z$  is introduced (Fig. 1), with the coordinate origin at the midpoint of the gap between the disks. The equation for the temperature of the medium, taking account of angular symmetry in steady motion, is as follows

$$\tilde{\kappa} \left( \frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) = -\text{tr} \sigma \cdot e. \quad (11)$$

The boundary conditions with motion of the medium between the disks (with no internal aperture) are: for the temperature:

$$T|_{z=-h} = T|_{z=h} = T_0, \quad T|_{r=R_1} = T_0, \quad \left. \frac{\partial T}{\partial r} \right|_{r=0} = 0, \quad (12)$$

for the velocity:

$$v|_{z=-h} = 0, \quad v|_{z=h} = \Omega r. \quad (13)$$

For the stress, the boundary conditions are discussed below. Note that the fourth condition in Eq. (12) is used in the case where the aperture  $R_2$  is small (Fig. 1), and its influence on the motion may be neglected.

Since  $h \ll R_1$ , it may be assumed, at some distance from the external edge and symmetry axis of the disks, that the temperature variation over the coordinate  $z$  is considerably larger than that over the radius  $r$ , that is

$$\frac{\partial^2 T}{\partial z^2} \gg \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r}, \quad (14)$$

and the equation for the temperature of the medium takes the form

$$\tilde{\kappa} \frac{\partial^2 T}{\partial z^2} = -\text{tr} \sigma \cdot e. \quad (15)$$

The second and third boundary conditions in Eq. (12) for this equation are not significant. The solution of this problem, at the level of isotropic pressure, coincides with the solution of the problem of Couette motion between parallel plates (Sec. 2) at each fixed  $r$ . In particular, it follows from this that the rheological components of the stress  $\sigma_1$ ,  $\sigma_2$ ,  $\sigma_{12}$  and the dimensionless shear velocity  $\Gamma$  are single-valued functions

$$\Gamma_{is} = \Omega \theta_{(0)} r / 2h, \quad (16)$$

i.e., of the radius.

The satisfaction of Eq. (14) is tested from the solution  $T(r, z)$  obtained; see Eqs. (5), (7), (8), and (16). As noted above, this condition ceases to hold at the center and edge of the disks.

In the central zone ( $r \sim h$ ), the solution is not constructed (although an approximate solution is not difficult to construct). This need not be done when  $R_2 \leq h$ , with weak heating of the medium (in the region of comparison with experiment) and a mean temperature of the medium  $\langle T \rangle \approx T_0$ . In the case when  $R_2 > h$ , the central region ( $r < R_2$ ) need not be taken into account, because its influence on the change in pressure is eliminated in accordance with the experimental procedures (Sec. 1).

At the ends of the disk ( $R_1 - r \sim h$ ), even isothermal flow is distorted by the influence of the edge, which is difficult to take into account; therefore, accurate calculation of the flow for engineering purposes, taking account of self-heating of the medium at the edge, does not make sense. A fairly rough estimate of the influence of the edge on the polymer motion between the disks may be made. Equation (11) for the temperature of the medium is averaged over the gap, assuming that the motion at the edge is nevertheless close to shear motion. In dimensionless form

$$K(\rho) + \frac{\partial^2 \langle T^* \rangle}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \langle T^* \rangle}{\partial \rho} = -\Psi(\Gamma), \quad (17)$$

where

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†The third boundary condition in Eq. (12) is thus approximately satisfied.

$$T^* = m(T - T_0); \quad \langle T^* \rangle = \frac{1}{2} \int_{-1}^1 T^* d\xi; \quad \xi = z/h;$$

$$\rho = \frac{r}{R_1}; \quad K(\rho) = \frac{R_1^2}{h^2} \left. \frac{\partial T^*}{\partial \xi} \right|_{\xi=1}; \quad \Psi(\Gamma) = \delta_p \frac{\Gamma_{is}}{\Gamma};$$

$$\delta_p = \frac{\sigma_{z\varphi}(\Gamma, \beta_k) \Gamma m R_1^2}{\bar{\alpha} \theta_{(0)1}}.$$

In writing the function  $\Psi(\Gamma)$ , the mean expression for the deformation rate in Eq. (3) obtained taking account of the boundary conditions in Eq. (13) is used

$$\Gamma_{is}/\Gamma = \frac{1}{2} \int_{-1}^1 \exp T^* d\xi, \quad (18)$$

where  $\Gamma_{is}$  is the deformation rate in an isothermal process; see Eq. (16).

The dependence of the dimensionless deformation rate on the mean temperature  $\langle T^* \rangle$  may be taken in the following approximate form†

$$\Gamma_{is}/\Gamma \approx \exp \langle T^* \rangle. \quad (19)$$

In the case where  $T^* \ll 1$ , the functional dependence in Eq. (19) is asymptotically accurate:

$$\Gamma_{is}/\Gamma = 1 + \langle T^* \rangle. \quad (20)$$

Note also that - see Eq. (12) - when  $\rho = 1$

$$T^* = \langle T^* \rangle = 0, \quad K(1) = 0. \quad (21)$$

The solution in Eq. (21) in the vicinity of  $\rho = 1$  may be written as a power series in  $(1 - \rho)$ :

$$\langle T^* \rangle = t_1(1 - \rho) - t_2(1 - \rho)^2 + O[(1 - \rho)^3], \quad (22)$$

where  $t_1 \geq 0$ ,  $t_2 \geq 0$  are unknown constants. Retaining only the first two terms in Eq. (22), substituting Eq. (22), into Eq. (17), and taking account of Eqs. (20) and (21), it is found that

$$t_1 + 2t_2 = \delta_p [\Gamma_{is}(R_1)]. \quad (23)$$

Then the solution in Eq. (22) (denoted by a minus sign) is combined with the mean solution  $\langle T^* \rangle(\rho)|_+$  of Eq. (15) at some radius  $\rho = \rho^*$ :

$$\langle T^* \rangle|_+ = \langle T^* \rangle|_- = t_1(1 - \rho^*) - t_2(1 - \rho^*)^2,$$

$$\left. \frac{\partial \langle T^* \rangle}{\partial \rho} \right|_+ = \left. \frac{\partial \langle T^* \rangle}{\partial \rho} \right|_- = t_1 + 2t_2(1 - \rho^*). \quad (24)$$

Thus,  $t_1$ ,  $t_2$ , and  $\rho^*$  are determined from Eqs. (23) and (24). It follows from Eq. (20) - or approximate Eq. (10) - which is valid for both the combined solutions that  $\Gamma_+ = \Gamma_-$ .

Thus, the dimensionless deformation-rate function  $\Gamma(r)$  is known over all ranges of  $r$ . The rheological component of the stress tensor is calculated from the known  $\Gamma(r)$  according to Eq. (1). To determine the pressure, which is measured by a manometer (Fig. 1), the equilibrium equation of the medium is used (noninertial approximation)

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\varphi\varphi}}{r} = 0. \quad (25)$$

Assuming that (in the given experiments,  $P_2 = 0$ )

$$\sigma_{rr}|_{r=R_2} = P_2, \quad \sigma_{rr}|_{r=R_1} = P_1,$$

and integrating Eq. (25), it is found that

†Equation (19) is derived on the basis that, in the subsequent calculations,  $T^* < 3$ , and the exponential for  $0 \leq T^* < 3$  may be roughly approximated by a linear function  $1 + \zeta T^*$  ( $\zeta = \text{const}$ ).

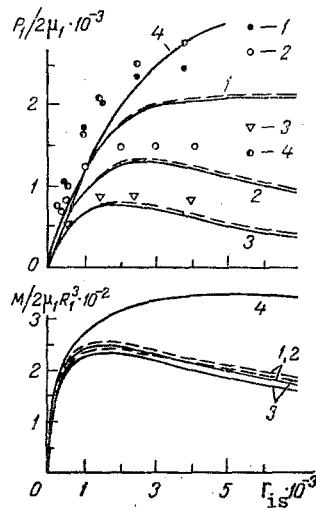


Fig. 4. Dependence of the pressure  $P_1/2\mu_1$  and moment  $M/2\mu_1R_1^3$  developing between parallel disks on the dimensionless deformation rate  $\Gamma_{is}(R_1)$  for flow with self-heating of the system with  $\Pi = 3.2 \cdot 10^{-6}$   $h/R_1 = 2.1 \cdot 10^{-2}$ : 1)  $R_2/R_1 = 2.5 \cdot 10^{-2}$ ; 2)  $1.75 \cdot 10^{-1}$ ; 3)  $3.75 \cdot 10^{-1}$  with thermostating of one disk; 4)  $R_2/R_1 = 2.5 \cdot 10^{-2}$  with thermostating of both disks; the points correspond to experiment and the curves to calculation.

$$P_1 = P_2 + \int_{R_2}^{R_1} \frac{\sigma_1 + \sigma_2}{r} dr. \quad (26)$$

The moment overcome in the rotation of a disk with an aperture is

$$M = 2\pi \int_{R_2}^{R_1} \sigma_{12} r dr. \quad (27)$$

Thus, using Eqs. (26) and (27), after the substitution of the corresponding expressions for the stress-tensor components from Eq. (1), it is sufficiently simple (for example, by the Simpson method) to determine the working characteristics of the seal (the pressure difference maintained and the frictional moment) from the known distribution of the dimensionless-deformation rate  $\Gamma$  with respect to the radius  $r$ .

In the region  $\rho < \rho^*$ , this distribution is sought numerically, by combined solution of Eqs. (7) and (8) using iterative methods. To obtain the analogous distribution  $\Gamma(r)$  in the region  $\rho > \rho^*$ , Eq. (22) for the distribution of the mean temperature over the radius must be used; this expression contains two unknown constants  $t_1$  and  $t_2$ . As a result, to find the complete solution of the whole problem, all that remains is to determine these constants and the value of  $\rho^*$ , which is simply accomplished by numerical solution of the system in Eqs. (23) and (24) using finite-difference and iterative methods.

As an example, the motion between disks for polyisobutylene melt  $MM = 5.7 \cdot 10^3$  is calculated numerically, with the following values of the constants:  $N = 2$ ,  $\theta_1 = 10$  sec;  $\theta_2 = 1.3 \cdot 10^{-1}$  sec;  $\mu_1 = 4.3 \cdot 10^2$  Pa;  $\mu_2 = 3 \cdot 10^3$  Pa;  $n = 3.2$ ;  $\alpha = 1.5 \cdot 10^{-1}$  W/m·deg;  $E = 14.2$  kcal/mole·deg ( $T = 22^\circ\text{C}$ ). The rheological constants are determined from isothermal experiments; see Fig. 2 and Eq. (1).

Experiment shows that there are groups of polymers in which the constants  $n$ ,  $\alpha_k$ , and  $\beta_k$  - see Eq. (9) - are approximately constant. Therefore, it is expedient to write the theoretical dependences (Fig. 4) in the form of  $P_1/P_2\mu_1$  ( $P_2 = 0$ ) and  $M/2\mu_1R_1^3$  on the dimensionless deformation rate of isothermal motion  $\Gamma_{is}(R_1) = \Omega R_1 \theta(0)_1 / 2h$  - see Eq. (8) - with fixed dimensionless parameters  $h/R_1$ ,  $R_2/R_1$ , and  $\Pi$  - see Eq. (9). The continuous curves in Fig. 4

show the dependences obtained without taking account of the edge,<sup>†</sup> and the dashed curves take it into account; see Eqs. (17)-(24). It follows from Fig. 4 that there is agreement between the theoretical and experimental data. Taking account of the edge in the given parameter range has practically no influence on the qualitative form of the curves, and weak quantitative influence. For more viscous liquids, where self-heating of the system under the same conditions is more considerable, the contribution of the stress from the moment at the edge to the pressure and the moment is increased.

It is evident from Fig. 4 that increase in  $R_2/R_1$  is associated with decrease in the pressure and moment with a fixed number of turns of the disk and more pronounced self-heating of the system (see the maximum on the curves). The latter occurs because  $\Gamma(\Gamma_{is})$  increases with increase in  $\Gamma_{is}$  (Fig. 3) with fixed radius  $r$  in the central region (where self-heating is insignificant), and decreases in the edge region (when self-heating is significant). This leads to the appearance of maxima on the curves of  $P_1/2\mu_1$  and  $M/2\mu_1R_1^3$  as a function of  $\Gamma_{is}(R_1)$  with sufficiently large  $R_2/R_1$ , because the relative contribution of the edge region increases.

With fixed geometric dimensions, pressure  $P_1/2\mu_1$  and moment  $M/2\mu_1R_1^3$ , and for sufficiently many rotations,  $\Gamma_{is}(R_1)$  is naturally larger in the case where both disks are thermostatted than in the case where one of the disks is thermally insulated (Fig. 4, curves and points 2, 4). Curves 2 and 4 coincide with small numbers of rotations, which indicates near-isothermal flow here.

Numerical calculations also show that, with a fixed number of rotations and fixed radii  $R_1$  and  $R_2$ , the pressure and moment sharply decrease with increase in the gap  $H$  and the temperature at the thermostatted surfaces  $T_0$ . In the first case,  $T_0$  is fixed; in the second,  $H$ .

#### NOTATION

$R_1$ , disk radius;  $R_2$ , radius of aperture at center of disk;  $H = 2h$ , thickness of gap between disks or plates;  $v_1 = v$ , plate velocity in Couette flow;  $T$ , temperature;  $T_0$ , temperature of thermostatted surface;  $\sigma, e$ , stress and deformation-rate tensors;  $\dot{\gamma}$ , shear velocity;  $\sigma_{ij}$ , components of stress tensor;  $\sigma_{12}$ , tangential component of stress tensor in shear;  $\sigma_1$  and  $\sigma_2$ , first and second differences of normal stress;  $c_k$ , internal parameter characterizing the elastic deformation in the system;  $\Gamma$ , dimensionless shear velocity;  $N$ , number of relaxational mechanisms in rheological model;  $\mu_k$ , elastic modulus;  $\theta_k$  and  $\theta_{(0)k}$ , relaxation times at temperatures  $T$  and  $T_0$ ;  $n$ , exponent of elastic potential;  $\Gamma_{is}$ , dimensionless deformation rate in isothermal process;  $\tilde{\kappa}$ , thermal conductivity;  $E$ , activation energy of viscous flow;  $R$ , universal gas constant;  $m = E/RT_0^2$ , constant;  $\Omega$ , angular velocity of disk rotation;  $r, \varphi, z$ , cylindrical coordinates;  $\rho$ , dimensionless current radius;  $x_1, x_2, x_3$ , rectangular coordinates;  $T^* = m(T - T_0)$ , dimensionless temperature;  $t_1$  and  $t_2$ , constants;  $\rho^*$ , radius at which the solutions are combined;  $P_1$  and  $P_2$ , pressure at edges of disks;  $M$ , moment at disks;  $\eta$ , greatest Newtonian viscosity;  $v_1$ , velocity of medium along axis  $x_1$ ;  $k$ , subscript denoting quantities in  $k$ -th Maxwellian relaxational mechanism;  $e_p$ , irreversible-deformation rate tensor.

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<sup>†</sup>The equation  $\tilde{\kappa} \frac{\partial^2 T}{\partial z^2} = -\sigma_{rz}(r) \dot{\gamma}(r)$  is solved with the conditions  $T|_{z=-h} = T|_{z=h} = T_0$  and those in Eq. (13).

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#### LIMITING HEAT TRANSFER IN HORIZONTAL TWO-PHASE THERMOSIPHON

M. K. Bezrodnyi and V. M. Podgoretskii

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Experimental results on the heat-transfer crisis in a horizontal thermosiphon with steam heating are outlined.

The wide use of autonomous heat-transfer devices - closed two-phase thermosiphons - entails comprehensive investigation of their characteristics in different operating conditions. The experimental material accumulated on the thermal and hydrodynamic characteristics of thermosiphons is basically related to the specific conditions of heat supply corresponding to boundary conditions of heat supply corresponding to boundary conditions of the second kind, which are modeled by means of electrical heating. Such rigorous heating conditions do not allow reliable experimental data to be obtained on the maximum heat-transfer capability in investigating inclined and horizontal thermosiphons [1-3], since this leads to premature heating of the evaporator wall along the upper generatrix on account of the stratification of the two-phase flow. To obtain experimental data on the limiting operating conditions of horizontal thermosiphons, investigations are undertaken with heat-supply boundary conditions of the third kind: steam heating.

The experimental apparatus (Fig. 1) includes: steam-generator 1, steam-heating chamber 2, experimental thermosiphon 3, and heat exchanger 4. The steam generator consists of a vertical tube 5 in which heat is liberated on account of direct transmission of a constant current and steam is generated, separation chamber 6, and external discharge channel 7. Heat supply to the thermosiphon evaporator is ensured on account of condensation of the steam ob-

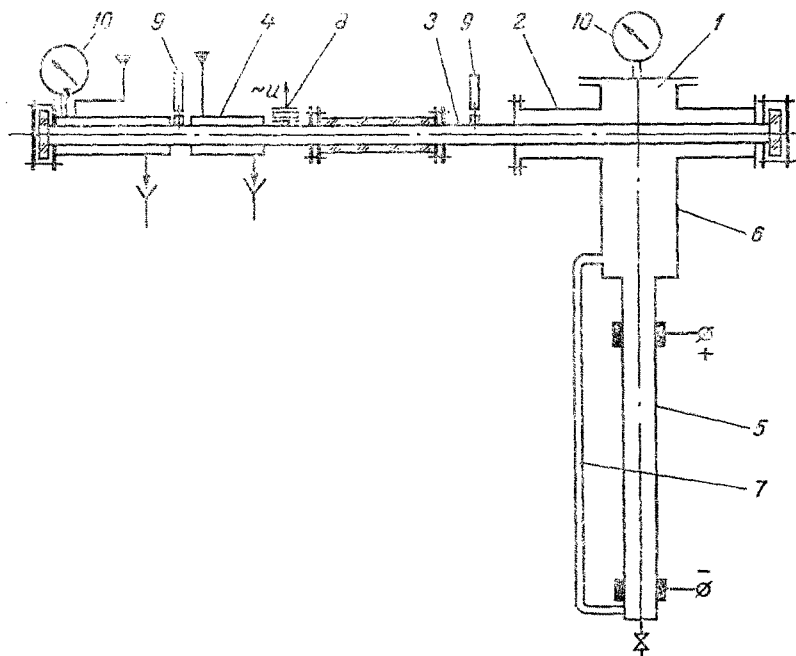


Fig. 1. Experimental apparatus.

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